Edge-pancyclicity and edge-bipancyclicity of faulty folded hypercubes

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Abstract

Let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the folded hypercube FQ_n so that $|F_v| + |F_e| \leq n - 2$, for $n \geq 2$. Choose any fault-free edge e. If $n \geq 3$ then there is a fault-free cycle of length l in FQ_n containing e, for every even l ranging from 4 to $2^n - 2|F_v|$; if $n \geq 2$ is even then there is a fault-free cycle of length l in FQ_n containing e, for every odd l ranging from $n + 1$ to $2^n - 2|F_v| - 1$.
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1 Introduction

Choosing an appropriate *interconnection network* (*network* for short) is an important integral part of designing parallel processing and distributed systems. There are a large number of network topologies that have been proposed. Among the proposed network topologies, the *hypercube* [1] is a well-known network model which has several excellent properties, such as recursive structure, regularity, symmetry, small diameter, short mean internode distance, low degree, and small edge complexity. Numerous variants of the hypercube have been proposed in the literature [3,4,17]. One variant that has been the focus of a great deal of research is the *folded hypercube*, which can be constructed from a hypercube by adding an edge joining every pair of vertices that are the farthest apart, i.e., two vertices with complementary addresses. The folded hypercube has been shown to be able to improve a

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system's performance over a regular hypercube in many measurements, such as diameter, fault diameter, connectivity, and so on [3, 20].

Since vertices and/or edges in a network may fail accidentally, it is necessary to consider the fault-tolerance of a network. Hence, the issue of fault-tolerant cycle embedding in an n-dimensional folded hypercube FQ_n has been studied in [2, 5, 7–14, 19, 20]. Embedding cycles in networks is important as many network algorithms utilize cycles as data structure. In this paper, let F_v and F_e be the sets of faulty vertices and faulty edges, respectively, in FQ_n. Choose any fault-free edge e. We prove that if $n \geq 3$, there is a fault-free cycle of length l in FQ_n containing e, for every even l ranging from 4 to $2^n - 2|F_v|$; if $n \geq 2$ is even then there is a fault-free cycle of length l in FQ_n containing e, for every odd l ranging from $n + 1$ to $2^n - 2|F_v| - 1$.

Throughout this paper, a number of terms—network and graph, node and vertex, edge and link—are used interchangeably. The remainder of this paper is organized as follows. In Section 2, we provide some necessary definitions and notations, and we present our main result in Section 3. Some concluding remarks are given in Section 4.

2 Basic definitions

A *path* in a graph $G = (V, E)$ is a sequence of distinct vertices so that any two consecutive vertices are joined by an edge, and the *length* of a path is the number of edges in the path. A *cycle* is a path of length at least 3 so that there is an edge joining the first and last vertices of the path, and the *length* of a cycle is the number of vertices in the cycle. For any graph $G = (V, E)$ and vertices $u, v \in V$, we denote the length of a shortest path in G from u to v by $d_G(u, v)$ (if there exists no path from u to v in G then $d_G(u, v)$ is defined as ∞). If C is a cycle of length c in the graph G containing the edge (x, y) and P is a path of length p in G from x to y that contains no vertices of C apart from x and y then we say that the cycle of length $c - 1 + p$ obtained by removing the edge (x, y) and including the path P is obtained by *grafting* the path P onto the cycle C. Let X be a set of vertices and edges of G. We denote the subgraph of G induced by the vertices of X and the vertices incident with the edges of X by $\langle X \rangle$. All other standard graph-theoretic terminology can be obtained from [21].

If a graph $G = (V, E)$ contains cycles of every length from 3 to $|V|$, then it is *pancyclic*, and it is *bipancyclic* if it contains a cycle of every even length from 4 to $|V|$, where $|V|$ denotes the number of vertices in G[1]. The pancyclicity is an important measurement of whether a network is suitable for an application inquiring cycles of any length within the network [6]. In a heterogeneous computing system, each edge and each vertex may be assigned with distinct computing power and distinct bandwidth, respectively [18]. Thus, it is worthwhile to extend pancyclicity to edge-pancyclicity and vertex-pancyclicity. If every edge (or vertex) of G lies on a cycle of every length from 3 to $|V|$ then G is said to be *edge-pancyclic* (or *vertex-pancyclic*), and G is *edge-bipancyclic* (or *vertex-bipancyclic*) if every edge (or vertex) lies on a cycle of every even length from 4 to $|V|$.

[1] The size of any set X of vertices and edges in a graph is denoted $|X|$.

We study graphs $G = (V, E)$ which model interconnection networks in which there might be faulty nodes or faulty links. Such faults are modelled by *faulty vertices* in V, the set of which we denote by F_v, and *faulty edges* in E, the set of which we denote by F_e. Every vertex of $V \setminus F_v$ is called *fault-free* and every edge of $E \setminus F_e$ that is not incident with any vertex of F_v is called *fault-free* (so, any fault-free edge is, by definition, incident only with fault-free vertices). If H is a subgraph of G then $(F_v \cup F_e) \cap H$ denotes the set of vertices of F_v and edges of F_e that lie in H. We say that a cycle or a path in G is *fault-free* if every vertex and edge that lies on the cycle or path is fault-free.

The *hypercube* Q_n has $\{0, 1\}^n$ as its vertex set and there is an edge joining two vertices if the vertex names differ in exactly one bit. The *folded hypercube of dimension* n, FQ_n, also has $\{0, 1\}^n$ as its vertex set. In FQ_n, there is an edge joining two vertices if the vertex names differ in exactly one bit or in every bit. If an edge is such that the two incident vertices differ in only the ith bit, for some $i \in \{1, 2, \ldots, n\}$, then we say that this edge lies in *dimension* i, with the neighbour of a vertex x where the edge lies in dimension i denoted $x^{(i)}$ (this applies to both Q_n and FQ_n); and if an edge is such that the two incident vertices differ in every bit then the edge is called a *complementary edge*, with the neighbour of a vertex x where the edge is a complementary edge denoted $\overline{x}$ (this applies only in FQ_n). Note that it makes sense to write, for example, $x^{(i,j)}$, to denote the vertex obtained by flipping the ith and jth bits of the name of x, and to write, for example, $\overline{x^{(i)}}$ to denote the vertex obtained by flipping every bit of the name of x except the ith. Consequently, the folded hypercube FQ_n is simply the hypercube Q_n with the addition of the complementary edges.

For FQ_n, we can choose some $i \in \{1, 2, \ldots, n\}$ and *partition* the folded hypercube over dimension i by separating the vertices whose ith component of their names is 0 from those whose ith component is 1. This results in two hypercubes of dimension $n-1$, denoted $Q_{n-1}^{0,i}$ and $Q_{n-1}^{1,i}$, induced by the vertices whose ith bits are 0 and 1, respectively. We suppress the superscript i if the partition dimension is understood. Of course, the complementary edges of FQ_n form a perfect matching, each incident with exactly one vertex in each hypercube, as do the edges of FQ_n lying in dimension i.

The folded hypercube FQ_n is clearly regular of degree $n+1$ and is known to be $(n+1)$-connected, vertex-transitive and edge-transitive [16, 19]. Both Q_n and FQ_n have been extensively studied. In particular, we shall use the following results.

Lemma 1 ([15])**.** *Let $n \geq 3$. Let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the hypercube Q_n so that $|F_v| + |F_e| \leq n-2$. Let u and v be any two distinct fault-free vertices in Q_n. There is a fault-free path of length l in Q_n joining u and v, for every l ranging from $d_{Q_n}(u, v) + 2$ to $2^n - 2|F_v| - 1$ where $l - d_{Q_n}(u, v)$ is even.*

Lemma 2 ([6])**.** *Let $n \geq 3$. Let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the hypercube Q_n so that $|F_v| + |F_e| \leq n - 2$. Choose any fault-free edge e. There is a fault-free cycle of length l in Q_n containing e, for every even l ranging from 4 to $2^n - 2|F_v|$.*

Lemma 3 ([19])**.** *Let $n \geq 2$. Let F_e be a set of faulty edges in the folded hypercube FQ_n so that $|F_e| \leq n - 1$. Choose any fault-free edge e. If $n \geq 3$ then there is a fault-free cycle* *of length l in FQ_n containing e, for every even l ranging from 4 to 2^n. If $n \geq 2$ is even then there is a fault-free cycle of length l in FQ_n containing e, for every odd l ranging from $n+1$ to $2^n - 1$.*

Lemma 4 ([2])**.** *Let $n \geq 2$. Let F_v be a set of faulty vertices in the folded hypercube FQ_n so that $|F_v| \leq n-2$. Choose any fault-free edge e. If $n \geq 3$ then there is a fault-free cycle in FQ_n containing e of length l, for every even l ranging from 4 to $2^n - 2|F_v|$. If $n \geq 2$ is even then there is a fault-free cycle in FQ_n containing e of length l, for every odd l ranging from $n+1$ to $2^n - 2|F_v| - 1$.*

Lemma 5 ([19])**.** *Let $n \geq 2$ and choose any edge e in FQ_n. The edge e lies on n cycles of length $n+1$ where the only edge appearing on more than one of these cycles is e.*

3 Main results

Proposition 6. *Let $n \geq 3$. Let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the folded hypercube FQ_n so that $|F_v| + |F_e| \leq n-2$. Choose any fault-free edge e. There is a fault-free cycle of length l in FQ_n containing e, for every even l ranging from 4 to $2^n - 2|F_v|$.*

Proof. If either $F_v = \emptyset$ or $F_e = \emptyset$ then the result follows by either Lemma 3 or Lemma 4, respectively. When $n = 3$, at least one of these conditions holds and so we are done. Henceforth, we assume that $n \geq 4$, $1 \leq |F_v| \leq n-3$ and $1 \leq |F_e| \leq n-3$.

Let $e = (u, v)$ be a fault-free edge. By [16,19], FQ_n is edge-transitive and so w.l.o.g. we may assume that u is named $0\ldots0000$ and v is named $0\ldots0001$ (that is, e lies in dimension n). Partition over some dimension that contains at least one edge of F_e; consequently, we obtain two hypercubes Q^0_{n-1} and Q^1_{n-1} where $|(F_v \cup F_e) \cap Q^i_{n-1}| \leq n-3$, for $i = 0, 1$. Define $F^i_v = Q^i_{n-1} \cap F_v$, for $i = 0, 1$. There are two cases: (1) the dimension we partition over is different to n; and (2) all faulty edges of F_e lie in dimension n and we partition over dimension n.

Case 1: W.l.o.g. we may assume that we have partitioned over dimension $n-1$. By Lemma 1, there is a fault-free cycle of length l in Q^0_{n-1} containing e, for every even l ranging from 4 to $2^{n-1} - 2|F^0_v|$. Choose such a cycle C of length $2^{n-1} - 2|F^0_v|$. As $2^{n-1} - 2|F^0_v| \geq 2^{n-1} - 2(n-3) \geq 2(n-2) + 2$, there is an edge (x, y) of C such that $(x, y) \neq e$ and all the edges of $\{(x, x^{(n-1)}), (x^{(n-1)}, y^{(n-1)}), (y, y^{(n-1)})\}$ are fault-free. Grafting the fault-free path $\langle x, x^{(n-1)}, y^{(n-1)}, y\rangle$ onto C yields a cycle C' of length $2^{n-1} - 2|F^0_v| + 2$. By Lemma 1, there is a fault-free path in Q^1_{n-1} joining $x^{(n-1)}$ and $y^{(n-1)}$ of length l', for every odd l' ranging from 3 to $2^{n-1} - 2|F^1_v| - 1$. Grafting the appropriate path onto C' yields the result.

Case 2: There exists a neighbour x of u in Q^0_{n-1} so that the path $\langle u, x, x^{(n)}, v\rangle$ is fault-free. This yields a fault-free cycle C containing e of length 4. By Lemma 1 applied to the edge (u, x) in Q^0_{n-1} and also to the edge $(v, x^{(n)})$ in Q^1_{n-1}, we can graft appropriate paths onto C so as to obtain the result. □

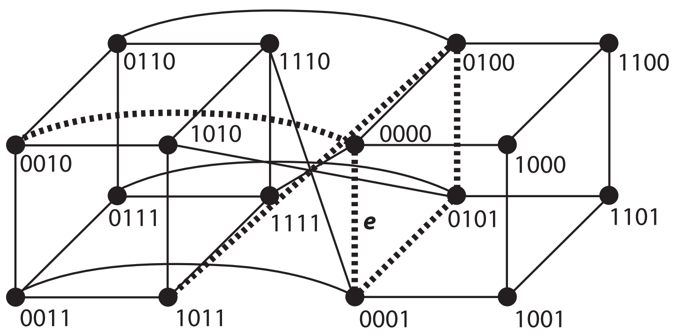

Figure 1: The folded hypercube FQ_4.

Lemma 7. *Let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the folded hypercube FQ_4 so that $|F_v|+|F_e| \leq 2$. Choose any fault-free edge in FQ_4. There is a fault-free cycle of length l in FQ_4 containing e, for every odd l ranging from 5 to $15-2|F_v|$.*

Proof. If $F_v = \emptyset$ (resp. $F_e = \emptyset$) then the result holds by Lemma 3 (resp. Lemma 4). So, suppose henceforth that $|F_v| = |F_e| = 1$.

Let $e = (u, v)$ be a fault-free edge. By Lemma 5, we obtain a cycle as required of length 5. By [16, 19], FQ_n is edge-transitive and so w.l.o.g. we may assume that $u = 0000$ and $v = 0001$. Partition over the dimension that contains the edge of F_e; consequently, we obtain two hypercubes Q_3^0 and Q_3^1 where one of the hypercubes contains the vertex of F_v and otherwise there are no faults in either hypercube. There are three cases: (1) Q_3^0 contains the edge e and the vertex of F_v; (2) Q_3^0 contains the edge e but not the vertex of F_v; and (3) neither Q_3^0 nor Q_3^1 contains the edge e.

Case 1: Suppose that Q_3^0 contains the edge e and the vertex of F_v. W.l.o.g., we may assume that we have partitioned over dimension 3 in order to get Q_3^0 and Q_3^1. The edge e lies on a fault-free cycle of length 4 in Q_3^0 and so, w.l.o.g., we may assume that 0100 and 0101 are fault-free. Also, either both edges of $\{(0000, 0010), (0100, 1011)\}$ are fault-free or both edges of $\{(0000, 1111), (0100, 0110)\}$ are fault-free; w.l.o.g. suppose that the edges of $\{(0000, 0010), (0100, 1011)\}$ are fault-free (the alternative yields an identical configuration). Hence, we have a fault-free path of length 5 from 0010 to 1011 that contains e. This path can be visualized in Fig. 1 (not all dimension-3 and complementary edges are shown). Hence, by choosing appropriate paths in (the fault-free) Q_3^1, we can clearly obtain fault-free cycles of lengths 7, 9 and 11 in FQ_4 containing e.

Let C_{11} be the cycle of length 11 constructed above. If $1101 \in F_v$ then we can replace the sub-path $\langle 0100, 0101, 0001 \rangle$ of C_{11} with the fault-free path $\langle 0100, 1100, 1000, 1001, 0001 \rangle$ to obtain a fault-free cycle containing e of length 13. If 1101 is fault-free then either the path $\langle 0100, 1100, 1101, 0101 \rangle$ is fault-free or the path $\langle 0101, 1101, 1001, 0001 \rangle$ is fault-free. Whichever is the case, we can graft the appropriate path onto C_{11} to obtain a fault-free cycle containing e of length 13.

Case 2: Suppose that Q_3^0 contains the edge e and Q_3^1 contains the vertex of F_v. W.l.o.g., we may assume that we have partitioned over dimension 3 in order to get Q_3^0 and Q_3^1. At least one of the following sets of edges contains only fault-free edges: $\{(0100, 1011), (0000, 0010)\}$;

$\{(0100, 0110), (0000, 1111)\}$; $\{(0101, 1010), (0001, 0011)\}$; and $\{(0101, 0111), (0001, 1110)\}$. W.l.o.g. suppose that the edges of $\{(0100, 1011), (0000, 0010\}$ are fault-free (the alternatives yield identical configurations). No matter which of the vertices of Q_3^1 is the vertex of F_v, we can easily obtain fault-free paths of lengths 2 and 4 from 1011 to 0010 in Q_3^1. By augmenting these paths with the edges of $\{(0100, 1011), (0000, 0010\}$ and the edge e, and then further augmenting these paths to build cycles using paths in (the fault-free) Q_3^0, we can clearly build fault-free cycles of lengths 7, 9, 11 and 13 in FQ_4 containing e.

Case 3: Suppose that neither Q_3^0 nor Q_3^1 contains the edge e; that is, we have partitioned over dimension 4 in order to get Q_3^0 and Q_3^1. W.l.o.g. we may assume that Q_3^0 is fault-free. At least one neighbour of 0000 is such that its incident complementary edge is fault-free; w.l.o.g. suppose that $(0010, 1101)$ is fault-free. No matter where the vertex of F_v lies in Q_3^1, we can find fault-free paths of lengths 2 and 4 from 0001 to 1101 in Q_3^1. Thus, this yields fault-free cycles of lengths 5 and 7 containing e. By grafting appropriate fault-free paths from 0000 to 0010 in Q_3^0 of lengths 3, 5 and 7 onto these cycles we obtain fault-free cycles of the required lengths containing e. □

Proposition 8. *Let $n \geq 2$ be even. Let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the folded hypercube FQ_n so that $|F_v| + |F_e| \leq n - 2$. Choose any fault-free edge e. There is a fault-free cycle of length l in FQ_n containing e, for every odd l ranging from $n+1$ to $2^n - 2|F_v| - 1$.*

Proof. It is trivial to check that the result holds for $n = 2$, and, by Lemma 7, the result holds for $n = 4$. If $F_v = \emptyset$ (resp. $F_e = \emptyset$) then the result follows by Lemma 3 (resp. Lemma 4). So, suppose henceforth that $n \geq 6$, $1 \leq |F_v| \leq n-3$ and $1 \leq |F_e| \leq n-3$.

Let $e = (u, v)$ be a fault-free edge. By Lemma 5, we obtain a cycle as required of length $n+1$; so we only have to worry about finding the required cycles of odd length ranging from $n+3$ to $2^n - 2|F_v| - 1$. By [16,19], FQ_n is edge-transitive and so w.l.o.g. we may assume that $u = 0\ldots0000$ and $v = 0\ldots0001$. Partition over some dimension that contains at least one edge of F_e; consequently, we obtain two hypercubes Q_{n-1}^0 and Q_{n-1}^1 where $|(F_v \cup F_e) \cap Q_{n-1}^i| \leq n-3$, for $i = 0, 1$. Define $F_v^i = Q_{n-1}^i \cap F_v$, for $i = 0, 1$. There are two cases: (1) the dimension we partition over is different to n; and (2) all faults of F_e lie in dimension n and we partition over dimension n.

Case 1: W.l.o.g. we may assume that we have partitioned over dimension $n-1$. Note that both u and v are incident with $n-2$ edges in Q_{n-1}^0 apart from e. For $i \in \{1, 2, \ldots, n-2\}$, define

- $S_i^{n-1} = \{(u, u^{(i)}), (u^{(i)}, u^{(i,n-1)})\}$
- $\overline{S_i} = \{(u, u^{(i)}), (u^{(i)}, \overline{u^{(i)}})\}$
- $T_i^{n-1} = \{(v, v^{(i)}), (v^{(i)}, v^{(i,n-1)})\}$
- $\overline{T_i} = \{(v, v^{(i)}), (v^{(i)}, \overline{v^{(i)}})\}$.

Note that because $n \geq 6$, $S_i^{n-1} \cup \overline{S_i}$ and $T_j^{n-1} \cup \overline{T_j}$ have no vertex nor edge in common, for any $i, j \in \{1, 2, \ldots, n-2\}$ (even if $i = j$).

As there are at most $n-2$ faulty vertices or edges in total and at least one faulty edge, w.l.o.g. we may assume that the edges of $S_i^{n-1} \cup \overline{T_j}$ are fault-free, for some $i, j \in \{1, 2, \ldots, n-2\}$ with $i \neq j$. Note that $d_{Q_{n-1}^1}(u^{(i,n-1)}, \overline{v^{(j)}}) = n-4 \geq 2$. By Lemma 1, there is a fault-free path of length l in Q_{n-1}^1 joining $u^{(i,n-1)}$ and $\overline{v^{(j)}}$, for every even l ranging from $n-2$ to $2^{n-1} - 2|F_v^1| - 2$. Hence, by augmenting any such path with the (fault-free) path $\langle u^{(i,n-1)}, u^{(i)}, u, v, v^{(j)}, \overline{v^{(j)}} \rangle$, we obtain a cycle of length l containing e, for every odd l ranging from $n+3$ to $2^{n-1} - 2|F_v^1| + 3$.

We now build a fault-free cycle of length l containing e, for all odd l ranging from $2^{n-1} - 2|F_v^1| + 5$ to $2^n - 2|F_v| - 1$. By Lemma 2, there is a fault-free cycle of length l' containing e in Q_{n-1}^0, for every even l' ranging from 4 to $2^{n-1} - 2|F_v^0|$. Choose such a cycle C of length l' where $l' \geq n$ (such a cycle exists as $2^{n-1} - 2(n-3) > n$ when $n \geq 6$). There exists an edge (x, y) on C such that $(x, y) \neq e$ and either $(x, x^{(n-1)})$ and $(y, \overline{y})$ are fault-free edges or $(x, \overline{x})$ and $(y, y^{(n-1)})$ are fault-free edges. W.l.o.g. suppose that $(x, x^{(n-1)})$ and $(y, \overline{y})$ are fault-free edges. Note that $d_{Q_{n-1}^1}(x^{(n-1)}, \overline{y}) = n-2$. By Lemma 1, there is a fault-free path of length l in Q_{n-1}^1 joining $x^{(n-1)}$ and $\overline{y}$, for every even l ranging from n to $2^{n-1} - 2|F_v^1| - 2$. Extend any such path with the edges $(x^{(n-1)}, x)$ and $(\overline{y}, y)$, and graft the resulting path onto the cycle C. Hence, there is a fault-free cycle of length $l' + l + 1$ containing e, for every even l' ranging from n to $2^{n-1} - 2|F_v^0|$ and for every even l ranging from n to $2^{n-1} - 2|F_v^1| - 2$; that is, there is a fault-free cycle of length l'' containing e, for every odd l'' ranging from $2n+1$ to $2^n - 2|F_v| - 1$. As $2^{n-1} - 2|F_v^1| + 3 \geq 2^{n-1} - 2(n-3) + 3 \geq 2n+1$ when $n \geq 6$, the result follows.

Case 2: As u is incident with $n-1$ edges in Q_{n-1}^0, there is a fault-free neighbour x of u so that the path $\langle u, x, \overline{x} \rangle$ is fault-free. W.l.o.g. we may assume that (u, x) lies in dimension $n-1$. Note that $d_{Q_{n-1}^1}(\overline{x}, v) = n-2$. By Lemma 1, there is a fault-free path of length l in Q_{n-1}^1 joining $\overline{x}$ and v, for every even l ranging from n to $2^{n-1} - 2|F_v^1| - 2$. Hence, by augmenting any such path with the (fault-free) path $\langle v, u, x, \overline{x} \rangle$, we obtain a cycle of length l containing e, for every odd l ranging from $n+3$ to $2^{n-1} - 2|F_v^1| + 1$.

Consider the edge (u, x). By Lemma 2, there is a fault-free path of length l' in Q_{n-1}^0 joining u and x, for every odd l' ranging from 3 to $2^{n-1} - 2|F_v^0| - 1$. Hence, by choosing the cycle of length $2^{n-1} - 2|F_v^1| + 1$ constructed in the previous paragraph and grafting such a path onto it, we obtain a cycle of length l containing e, for every odd l ranging from $2^{n-1} - 2|F_v^1| + 3$ to $2^n - 2|F_v| - 1$. The result follows. □

Theorem 9. *Let $n \geq 2$. Let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the folded hypercube FQ_n so that $|F_v| + |F_e| \leq n-2$. Choose any fault-free edge e. If $n \geq 3$ then there is a fault-free cycle of length l in FQ_n containing e, for every even l ranging from 4 to $2^n - 2|F_v|$. If $n \geq 2$ is even then there is a fault-free cycle of length l in FQ_n containing e, for every odd l ranging from $n+1$ to $2^n - 2|F_v| - 1$.*

4 Concluding Remarks

Fault-tolerance is an increasingly important research topic in the area of the multi-processor computer systems, and many studies have focused on the vertex fault-tolerant or edge fault-tolerant properties of some specific networks. In this paper, let F_v and F_e be sets of faulty vertices and faulty edges, respectively, in the folded hypercube FQ_n so that $|F_v| + |F_e| \leq n - 2$, for $n \geq 2$. Choose any fault-free edge e. If $n \geq 3$ then there is a fault-free cycle of length l in FQ_n containing e, for every even l ranging from 4 to $2^n - 2|F_v|$; if $n \geq 2$ is even then there is a fault-free cycle of length l in FQ_n containing e, for every odd l ranging from $n+1$ to $2^n - 2|F_v| - 1$. Our results strengthen the possibilities of using folded hypercubes in interconnection networks where fault-tolerance is important.

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